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Comparison of the Two-Point Method and All-Time-Step Advance Approach to Estimate Infiltration Parameters for Surface Irrigation

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ABSTRACT

Background and Objectives: Surface irrigation remains the most prevalent irrigation method globally, particularly in regions with limited access to advanced technology, due to its operational simplicity, low infrastructure costs, and minimal energy demands. Despite its widespread use, the water application efficiency of surface irrigation systems is frequently undermined by inaccuracies in estimating hydraulic parameters, most notably infiltration coefficients. These coefficients, which quantify the rate at which water penetrates soil over time, are foundational to designing irrigation systems that balance water application with soil absorption. Accurate infiltration modelling is critical to minimizing environmental and agronomic challenges such as surface runoff (which wastes water and transports nutrients) and deep percolation (which depletes groundwater and leaches fertilizers).

Traditional estimation techniques, such as the two-point method proposed by Elliott and Walker (1982), derive infiltration parameters using data from two discrete phases: when water reaches the midpoint and the endpoint of the field. While this method is computationally efficient and field-friendly, it oversimplifies the infiltration process by ignoring temporal variability, such as fluctuations in soil moisture, micro-topography, or hydraulic resistance, that occur during irrigation. To address this limitation, this study introduces an all-time-step approach (Method 1), which integrates flow depth measurements from every advanced phase along the field. By leveraging continuous data from all stations, Method 1 captures dynamic infiltration patterns, enabling more robust calibration of the Kostiakov and Kostiakov-Lewis equations. The primary objective of this study is to rigorously compare the accuracy of the traditional two-point method with a novel all-time-step approach for determining infiltration coefficients in surface irrigation. The innovation lies in the development of the all-time-step method, which leverages continuous flow depth measurements from every advanced phase along the field, capturing dynamic infiltration patterns that are often overlooked by traditional methods. This approach significantly improves the calibration of the Kostiakov and Kostiakov-Lewis equations, leading to improved irrigation

scheduling, optimized water-use efficiency, and enhanced support for sustainable agricultural practices.

Materials and Methods: The study utilized experimental data from five open-ended furrows evaluated by Ramsey (1976) at the University of Arizona's research farm, a semi-arid site characterized by sandy loam soils. These furrows, designed to replicate real-world agricultural conditions, spanned up to 330 ft (100.6 m) and were monitored under variable hydraulic parameters. Key measured variables included furrow geometry (cross-sectional profiles and longitudinal slopes of 0.1–0.1068%), inflow rates (0.023–0.065 m^3/s adjusted to simulate diverse irrigation scenarios), and flow depths recorded at 30-foot (9.1 m) intervals during both the advance phase (when water advance reaches the downstream end of the field) and storage phase.

Surface storage volumes were calculated using the trapezoidal method, a numerical integration technique that divides the furrow into discrete segments and computes cumulative cross-sectional areas across all stations. This approach accounts for irregular furrow shapes and spatial variations in flow depth, ensuring precise estimation of stored water. Infiltrated volumes were derived by subtracting surface storage from the total inflow volume, while average infiltration depth and opportunity time (the duration water remains in contact with the soil at each station) were computed to calibrate the Kostiakov and Kostiakov-Lewis models.

Two calibration methodologies were rigorously evaluated:
Method 1 (All Advanced Time Steps): Leveraged nonlinear regression in Microsoft Excel to optimize model coefficients using data from all advanced phases (11 time steps per furrow), ensuring high temporal resolution.
Method 2 (Two-Point or Two Advance Time Steps): Implemented via WinSRFR software (version 5.1.1), this approach relied on midpoint and endpoint advance data with volume balance equations, adhering to Elliott and Walker's (1982) methodology.

Model performance was assessed using Nash-Sutcliffe Efficiency (*NSE*) (predictive accuracy), coefficient of determination (R^2), Mean Absolute Percentage Error (*MAPE*), and boxplot analysis. These metrics evaluated deviations between observed and predicted infiltration depths across advance and storage phases, emphasizing practical applicability in irrigation planning. For instance, boxplots highlighted systematic overestimation trends, guiding adjustments for field-specific conditions like soil heterogeneity or slope variability.

Results: The Kostiakov-Lewis equation derived via Method 1 demonstrated superior accuracy in estimating average infiltration depth, achieving a *MAPE* of 12.6%, significantly lower than the TP method's 26.1%. For instance, in Furrow 5, Method 1 reduced *MAPE* to 7.3%, highlighting its precision. In contrast, the Kostiakov model under Method 1 exhibited higher errors (*MAPE* = 44.3%), underscoring the importance of the steady-term parameter in the Kostiakov-Lewis equation for short-term advance-phase dynamics. During the storage phase, however, the Kostiakov model (Method 1) outperformed others, with a median absolute error of 10.8 $mm.m^2/m$ compared to 14.7 $mm.m^2/m$ for the TP method, suggesting its adaptability to prolonged infiltration.

Statistical evaluation revealed that the Kostiakov-Lewis model (Method 1) consistently achieved *NSE* values exceeding 0.7 across all furrows, indicating "good to excellent" alignment with field data. The

Kostiakov model (Method 1) showed moderate performance, with *NSE* ranging from 0.66 to 0.96. Box plot analysis further exposed systematic overestimation of infiltration depths by all models, though Method 1's Kostiakov equation yielded the lowest median error (10.9 mm.m²/m), making it reliable for operational irrigation planning. Contradictions emerged in model performance: while the Kostiakov-Lewis equation excelled in advance-phase accuracy, it lagged in storage-phase simulations, whereas the simpler Kostiakov model demonstrated versatility across phases, emphasizing the need for context-specific model selection.

Conclusion: Based on the hypothetical point estimation, the Kostiakov-Lewis infiltration equation derived from the all-time-steps method had the highest accuracy in estimating the average depth of infiltrated water during the average opportunity time. However, based on the estimation of infiltrated water depths during the storage phase, the Kostiakov equation derived from the same method showed the highest accuracy. Therefore, the equation with the least error in estimating the average depth of farm infiltrated water does not necessarily have the highest accuracy in farm irrigation planning. Using all time steps of the advanced phase to determine the infiltration relationship can lead to increased accuracy of the infiltration equation. In the present study, the Kostiakov infiltration equation derived from the all-time-steps method demonstrated the highest accuracy.

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مقایسه روش دو نقطه‌ای و تمامی گام‌های زمانی پیشروی در تعیین ضرایب نفوذ در آبیاری سطحی

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چکیده

زمینه و اهداف: آبیاری سطحی به‌دلیل سادگی عملکرد، هزینه‌های زیرساختی پایین و نیاز انرژی کم، هم‌چنان پرکاربردترین روش آبیاری در جهان است، به‌ویژه در مناطقی با دسترسی محدود به فناوری‌های پیشرفته. با این‌حال، کارایی کاربرد آب در سیستم‌های آبیاری سطحی اغلب به‌دلیل عدم برآورد دقیق پارامترهای هیدرولیکی، به‌ویژه ضرایب نفوذ، کاهش می‌یابد. این ضرایب که نرخ نفوذ آب به خاک را در طول زمان کمّی می‌کنند، پایه‌ای برای طراحی سیستم‌های آبیاری هستند تا تعادلی بین آب کاربردی و جذب خاک ایجاد شود. مدل‌سازی دقیق نفوذپذیری برای کاهش چالش‌های زیست‌محیطی و زراعی مانند رواناب سطحی (که موجب هدررفت آب و انتقال مواد مغذی می‌شود) و نفوذ عمقی (که منابع آب زیرزمینی را کاهش و کودها را شستشو می‌دهد) حیاتی است.

روش‌های سنتی برآورد نفوذ، مانند روش دو نقطه‌ای الیوت و واکر (۱۹۸۲)، پارامترهای نفوذ را با استفاده از داده‌های دو مرحله گسسته محاسبه می‌کنند: زمانی که آب به نقطه میانی و انتهای مزرعه می‌رسد. اگرچه این روش از نظر محاسباتی کارآمد و قابل اجرا در مزرعه است، اما فرآیند نفوذ را با نادیده گرفتن تغییرات زمانی مانند نوسانات رطوبت خاک، ریزتوپوگرافی یا مقاومت هیدرولیکی در طول آبیاری، بیش از حد ساده‌سازی می‌کند. برای رفع این محدودیت، این مطالعه روش تمام گام‌های زمانی (روش ۱) را معرفی می‌کند که اندازه‌گیری‌های عمق جریان را از تمام مراحل پیشروی آب در طول مزرعه ادغام می‌نماید. با استفاده از داده‌های پیوسته از تمام ایستگاه‌ها، روش ۱ الگوهای پویای نفوذ را شناسایی می‌کند و امکان کالیبراسیون دقیق‌تر معادلات کاستیاکوف و کاستیاکوف-لوئیس را فراهم می‌سازد.

هدف اصلی این مطالعه، مقایسه دقت روش سنتی دو نقطه‌ای با روش نوین تمام گام‌های زمانی در تعیین ضرایب نفوذپذیری در آبیاری سطحی است. نوآوری این پژوهش در توسعه روش تمام گام‌های زمانی نهفته است که با بهره‌گیری از اندازه‌گیری‌های پیوسته عمق جریان در تمام مراحل پیشروی آب، الگوهای پویای نفوذ را شناسایی می‌کند که اغلب توسط روش‌های سنتی نادیده گرفته می‌شوند. این رویکرد منجر به بهبود کالیبراسیون معادلات کاستیاکوف و کاستیاکوف- لوئیس، برنامه‌ریزی دقیق‌تر آبیاری، بهینه‌سازی کارایی مصرف آب و پشتیبانی بهتر از روش‌های کشاورزی پایدار می‌شود.

مواد و روش‌ها: این مطالعه از داده‌های مزرعه‌ای پنج جویچه با انتهای باز استفاده کرد که توسط رمزی (۱۹۷٦) در مزرعه تحقیقاتی دانشگاه آریزونا واقع در منطقه نیمه‌خشک با خاک لوم شنی، ارزیابی شده بودند. این جویچه‌ها که برای شبیه‌سازی شرایط واقعی کشاورزی طراحی شده بودند، تا طول ۳۳۰ فوت (۱۰۰/٦ متر) امتداد داشتند و تحت پارامترهای هیدرولیکی متغیر مورد پایش قرار گرفتند. متغیرهای کلیدی اندازه‌گیری شده شامل: هدسه جویچه (پرفایل سطح مقطع و شیب‌های طولی در محدوده ۰/۱–۰/۱۰٦۸ درصد)، دبی‌های ورودی (۰/۰۲۳–۰/۰٦۵ مترمکعب بر ثانیه که برای شبیه‌سازی سناریوهای مختلف آبیاری تنظیم شده بودند) و عمق جریان در فواصل ۳۰ فوتی (۹/۱ متری) در هر دو فاز پیشروی (زمانی که پیشروی آب به انتهای پایین‌دست مزرعه می‌رسد) و ذخیره ثبت شد.

حجم آب ذخیره شده سطحی با استفاده از روش ذوزنقه‌ای (یک تکنیک انتگرال‌گیری عددی که جویچه را به بخش‌های مجزا تقسیم و مساحت‌های مقطعی تجمعی را در تمام ایستگاه‌ها محاسبه می‌کند) محاسبه شد. این روش قادر به در نظر گرفتن اشکال نامنظم فارو و تغییرات مکانی عمق جریان است و تخمین دقیقی از آب ذخیره شده ارائه می‌دهد. حجم آب نفوذی از طریق تفاضل حجم ذخیره سطحی از حجم کل آب ورودی به‌دست آمد، در حالی‌که عمق متوسط نفوذ و فرصت زمان (مدت زمان تماس آب با خاک در هر ایستگاه) برای کالیبراسیون مدل‌های کاستیاکوف و کاستیاکوف- لوئیس محاسبه شدند. دو روش کالیبراسیون به دقت ارزیابی شدند:

روش ۱ (همه گام‌های زمانی پیشروی): از رگرسیون غیرخطی در Microsoft Excel برای بهینه‌سازی ضرایب مدل با استفاده از داده‌های تمام گام‌های پیشروی (۱۱ گام زمانی برای هر جویچه) استفاده کرد که وضوح زمانی بالایی را تضمین می‌نماید.

روش ۲ (دو نقطه‌ای یا دو گام زمانی پیشروی): که از طریق نرم‌افزار WinSRFR (نسخه ۵,۱,۱) پیاده‌سازی شد و بر داده‌های پیشروی در نقطه میانی و انتهایی با معادلات موازنه حجم تکیه داشت و از روششناسی الیوت و واکر (۱۹۸۲) پیروی می‌کرد.

دقت مدل‌ها با شاخص‌های کارایی نش- ساتکلیف (*NSE*)، ضریب تعیین (R^2)، درصد میانگین خطای مطلق (*MAPE*) و تحلیل نمودار جعبه‌ای (Boxplot) مقایسه شد. این معیارها انحرافات بین عمق‌های نفوذ مشاهده شده و پیش‌بینی‌شده را در مراحل پیشروی و ذخیره ارزیابی کردند و بر کاربرد عملی آن‌ها در برنامه‌ریزی آبیاری تأکید داشتند. به عنوان مثال، نمودارهای جعبه‌ای روندهای سیستماتیک بیش‌برآورد را نشان دادند که راهنمایی برای تنظیمات متناسب با شرایط خاص مزرعه مانند ناهمگنی خاک یا تغییرپذیری شیب بودند.

نتایج: معادله کاستیاکوف- لوئیس استخراج شده از روش ۱ دقت بالاتری در تخمین متوسط عمق نفوذ نشان داد و به متوسط درصد خطای مطلق (*MAPE*) برابر ۱۲/۶٪ دست یافت که به‌طور معناداری کم‌تر از مقدار ۲۶/۱٪ حاصل از روش دو نقطه‌ای بود. به عنوان مثال، در جویچه شماره ۵، روش ۱ توانست *MAPE* را تا ۷/۳٪ کاهش دهد که نشان‌دهنده دقت بالای این روش است. در مقابل، مدل کاستیاکوف در روش ۱ خطای بالاتری (*MAPE*=٪۴۴/۳) نشان داد که اهمیت پارامتر نفوذ نهایی در معادله کاستیاکوف- لوئیس را برای دینامیک مرحله پیشروی کوتاه‌مدت برجسته می سازد. با این‌حال، در مرحله ذخیره، مدل کاستیاکوف (روش ۱) عملکرد بهتری نسبت به سایر روش ها داشت و به میانه خطای مطلقی برابر $mm.m^2/m$ ۱۰/۸ در مقایسه با $mm.m^2/m$ ۱۴/۷ برای روش دو نقطه‌ای دست یافت که نشان‌دهنده تطابق‌پذیری بهتر این مدل با شرایط نفوذ طولانی‌مدت است.

ارزیابی آماری نشان داد که مدل کاستیاکوف-لوئیس (روش ۱) در تمامی فاروها مقادیر *NSE* بیش از ۰/۷ را به‌دست آورد که نشان‌دهنده همخوانی "خوب تا عالی" با داده‌های میدانی است. از سوی دیگر، مدل کاستیاکوف (روش ۱) عملکرد متوسطی داشت، با دامنه *NSE* بین ۰/۶۶ تا ۰/۹۶. تحلیل نمودار جعبه‌ای نیز نشان داد که تمامی مدل‌ها تمایل به تخمین بیش‌برآورد عمق نفوذ دارند، هرچند معادله کاستیاکوف در روش ۱ کم‌ترین میانه خطا ($mm.m^2/m$ ۱۰/۹) را تولید کرد و آن را به گزینه‌ای قابل اعتماد برای برنامه‌ریزی عملیاتی آبیاری تبدیل نمود. تناقضاتی در عملکرد مدل‌ها مشاهده شد: درحالی که معادله کاستیاکوف- لوئیس در دقت مرحله پیشروی برتری داشت، در شبیه‌سازی مرحله ذخیره عملکرد ضعیف‌تری نشان داد. در مقابل، مدل ساده‌تر کاستیاکوف در هر دو مرحله انعطاف‌پذیری بهتری داشت که لزوم انتخاب مدل بر اساس شرایط خاص را برجسته می‌سازد.

نتیجه‌گیری: بر اساس برآورد نقطه فرضی، معادلۀ نفوذ کاستیاکوف- لوئیس استخراج شده از روش تمامی گام‌های زمانی، بالاترین دقت را در تخمین میانگین عمق آب نفوذی در طول میانگین فرصت زمان نفوذ نشان داد. با این‌حال، در تخمین عمق آب نفوذی در مرحله ذخیره، معادله کاستیاکوف حاصل از همان روش بیش‌ترین دقت را داشت. بنابراین، معادله‌ای که کم‌ترین خطا را در تخمین میانگین عمق آب نفوذی مزرعه دارد، لزوماً از بالاترین دقت در برنامه‌ریزی آبیاری مزرعه برخوردار نیست. استفاده از تمامی گام‌های زمانی مرحله پیشروی برای تعیین رابطه نفوذ می‌تواند به افزایش دقت معادله نفوذ منجر شود. در مطالعه حاضر، معادله نفوذ کاستیاکوف استخراج شده از روش تمامی گام‌های زمانی بالاترین دقت را نشان داد.

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Introduction

Surface irrigation remains the most widely used irrigation method globally due to its simplicity, low cost, and minimal energy requirements. When properly designed, implemented, and managed, surface irrigation can achieve application efficiencies comparable to other irrigation systems (1, 2). However, one factor contributing to the lower efficiency of surface irrigation systems is the high uncertainty associated with their hydraulic parameters (3). Accurate determination of the infiltration relationship is crucial for optimizing the hydraulic performance of these systems, as it influences all hydraulic parameters, including infiltrated water volume, surface storage volume, advance time, irrigation duration, and application efficiency. In other words, inaccuracies in the infiltration equation can lead to non-uniform water distribution, increased surface runoff losses, increased deep percolation losses (4), and reduced crop yield (5). Therefore, employing an appropriate infiltration model along with a precise method for determining its empirical coefficients is of paramount importance in surface irrigation (6, 7).

Various mathematical models have been developed to describe the infiltration process in surface irrigation (8, 9). These models typically express the infiltration rate as a function of time, soil properties, and other environmental factors. One of the earliest models is the Kostiakov equation, which describes the infiltration rate decreasing over time following a power-law relationship (10). While widely used due to its simplicity, the Kostiakov model does not account for a nearly constant infiltration rate over extended periods. A modified version of this model is the Kostiakov-Lewis equation, which adds a constant infiltration rate term to the original Kostiakov equation (11). Another well-known model is the Philip equation, which is based on a more theoretical understanding of infiltration dynamics in porous media. The Philip equation describes infiltration as a combination of a time-dependent term and a steady-state infiltration term. This model is often applied to short-duration irrigation events (12). Other infiltration models include empirical relationships such as the Horton equation (13), which describes the decrease in infiltration rate over time due to soil saturation, and the Green-Ampt model (14), which is based on the physical principles of infiltration driven by capillary suction and gravitational forces. Each of these models has its strengths and limitations depending on soil type, irrigation duration, and field conditions. Therefore, no single equation is universally suitable for all soil types and irrigation scenarios. Selecting the most appropriate infiltration relationship depends on factors such as soil type, soil structure, irrigation method, and irrigation duration (15). Therefore, choosing the most appropriate model for designing surface irrigation systems requires careful consideration of local conditions.

After selecting an appropriate infiltration model, it is essential to accurately determine the empirical parameters. Various methods have been developed for estimating these coefficients, ranging from direct field measurements to indirect approaches. Indirect methods include using water advance and recession data along the field in conjunction with volume balance equations (16), dimensionless advance curve fitting (17), the one-point method by Shepard et al. (1993) (18), the one-point method by Valiantzas et al. (2001) (19), the multi-step calibration method by Walker (2005) (20), and newer methods recently proposed by Seyedzadeh et al. (2020a) (21), Seyedzadeh et al. (2020b) (22), Panahi et al. (2021) (23), Panahi et al. (2022) (24), and Panahi et al. (2023) (25). Each of these methods has its inherent limitations. One of the most significant challenges for methods based on advance and recession data is the accurate determination of surface water storage volume during irrigation, which plays a critical role in estimating infiltration coefficients. Many methods do not account for variations in surface flow depth and their impact on soil infiltration, leading to potential errors in estimating surface storage volume and, consequently, the infiltration coefficients (26).

The two-point method by Elliott and Walker (1982) is one of the most widely used and practical approaches for estimating infiltration parameters due to its simplicity and usually acceptable accuracy (17). This method, based on a volume balance principle, relies on data from two distinct time points during the irrigation process: the time that water reaches the midpoint of the field and then the downstream end of the field. In this method, the surface water storage volume is estimated at the two-time steps and compared with the total volume of water applied to the field to calculate the infiltrated water volume. The surface water storage volume is determined using the flow cross-section at the upstream end of the field and applying a shape factor. Subsequently, with the infiltrated water volume known at these two-time points, the coefficients of the infiltration equation are calculated. This process can be extended to all advance times along the length of the field. In other words, for every advance time step along the field, the surface storage volume can be estimated, and by comparing it with the applied water volume, the infiltrated water volume can be computed. This provides infiltration data at multiple points corresponding to each advance time step, rather than just two points, providing more comprehensive information for deriving the empirical parameters of the chosen infiltration equation.

Therefore, the primary objective of the present study was to compare the accuracy of infiltration relationships derived from the traditional two-point method with those obtained from a novel all-time-step approach. The innovation of this study lies in the development of this all-time-step method, which integrates flow depth measurements from every advance phase along the field. Unlike the two-point method, which relies on data from only two discrete time steps, the all-time-step approach captures the dynamic nature of infiltration processes more completely, including temporal variability in soil moisture, micro-topography, and hydraulic resistance. This method provides a more comprehensive and accurate estimation of infiltration coefficients, enabling better calibration of the Kostiakov and Kostiakov-Lewis equations.

By addressing the limitations of traditional methods, this study introduces a significant advancement in infiltration modelling for surface irrigation. The improved accuracy of the all-time-step approach has practical implications for irrigation planning, particularly in water-scarce regions, as it enables more precise irrigation scheduling, optimized water-use efficiency, and enhanced support for sustainable agricultural practices. These contributions represent a meaningful step forward in the field of surface irrigation and distinguish this study from previous research.

Materials and Methods

This study utilized data from the evaluation of five experimental furrows conducted by Ramsey (1976) at the research farm of the University of Arizona (27). All experiments involved open-ended furrows (i.e. free surface runoff), with lengths marked by stations at 30-foot intervals. During irrigation, the flow depth was measured at each station as the water reached it. Figure 1 illustrates the variations in flow depth recorded at different time steps for all furrows. Furthermore, after the water reached the end of the field, the water depth was measured instantaneously at all stations at specified time intervals. The water depth data from these stations were then used to determine the surface storage volume in this research. For further details, the source can be consulted. Some geometric characteristics of the furrows and hydraulic parameters are provided in Table 1.

Table 1. Some geometric characteristics of furrows and flow hydraulics in irrigation experiments by Ramsey (1976) (27).

Characteristic		Furrow No.				
		1	2	3	4	5
Furrow Length (ft)		300	330	330	330	330
Water advance time at the downstream end of the field (min)		29.22	17.95	17.05	15.65	17.04
Inflow discharge (m^3/s)		0.047	0.047	0.065 (t=0-102) 0.048 (t=102-168) 0.032 (t=168-220)	0.060 (t=0-294) 0.033 (t=294-352)	0.063 (t=0-108) 0.023 (t=108-197)
Cut-off Time (min)		50	208	220	352	197
Longitudinal slope (%)		0.1073	0.1032	0.0996	0.1068	0.1029
**$W_p = \gamma_1 y^{\gamma_2}$ (ft)	γ_1*	3.107	2.636	2.888	2.873	2.827
	γ_2*	0.5949	0.5005	0.5598	0.5468	0.5732
**$A = \sigma_1 y^{\sigma_2}$ (ft^2)	σ_1*	0.760	1.474	1.375	1.668	1.545
	σ_2*	1.2071	1.4429	1.4408	1.4975	1.5542
**$T = \alpha_1 y^{\alpha_2}$ (ft)	α_1*	2.710	2.095	2.392	2.350	2.311
	α_2*	0.5736	0.4539	0.5147	0.4912	0.5166

* These are constant coefficients determined using field data.
** The parameters T, A, and W_p represent the top width of the flow (m), the cross-sectional area of the flow (m^2), and the wetted perimeter of the furrow (m), respectively. These were derived based on field measurements of the furrow's cross-sectional shape.

Using the inflow and outflow data of the furrows, the final infiltration rate for each furrow was determined, and the values are presented in Table 2, in which the final infiltration rate for the first furrow could not be calculated due to the short duration of the irrigation event.

Table 2. Final infiltration rate determined using inflow and outflow discharge for tested furrows.

Parameter	Furrow No.				
	1	2	3	4	5
$f_o \left(mm.m^2/m.h\right)$	-	8.792	12.778	11.834	13.422

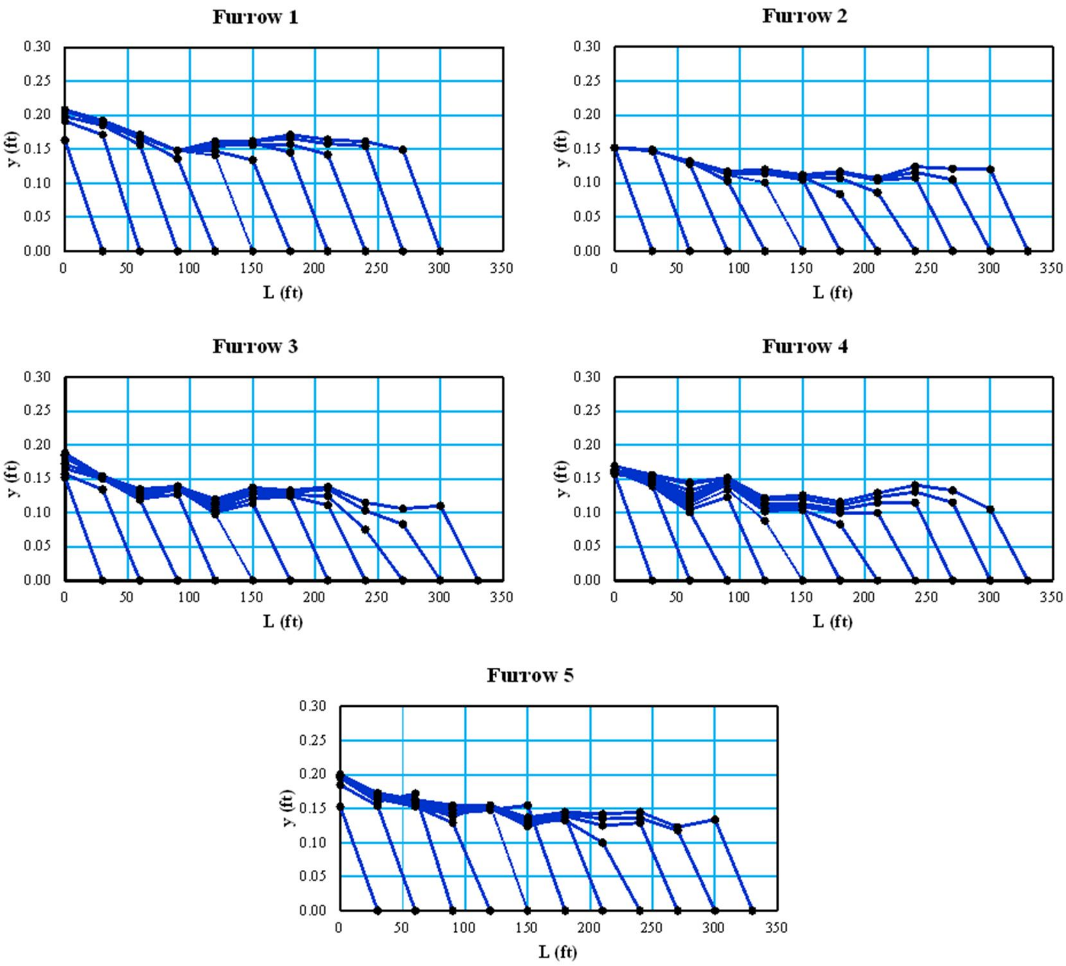

Figure 1. Changes in the surface water flow depth of tested furrows during the advance phase (27).

Methods for Extracting Infiltration Equation Coefficients

Method 1: Utilizing All-Advance-Time Steps

Given the availability of water depth data at all stations, it is possible to calculate the flow cross-sectional area at every station. Therefore, at each time step during the water advance along the field, the surface storage volume was computed using the trapezoidal integration method (Figure 2). As shown in Figure 2, and considering that the spacing between stations along the field is equal, the final equation for determining the surface storage volume is expressed as Eq. 1.

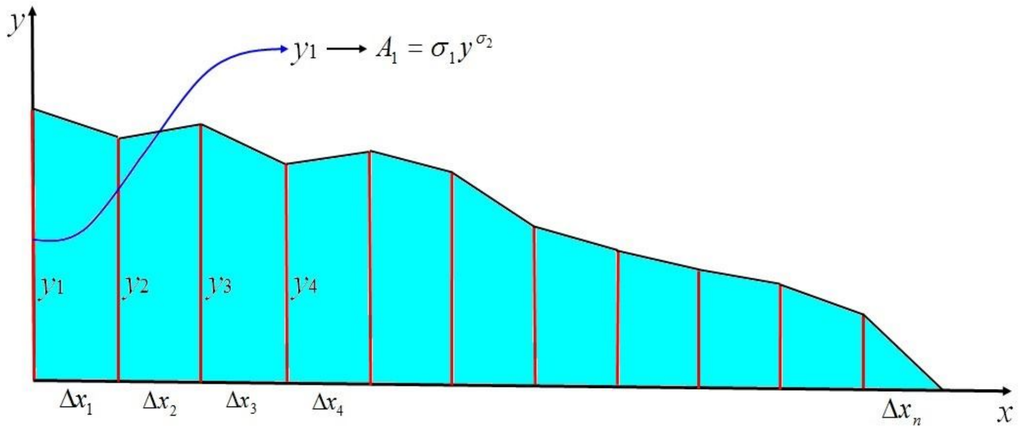

Figure 2. Surface flow profile and its division, for calculating the volume of surface water stored in surface irrigation (28).

$$V_{Surface-i} = \Delta x\left(\frac{A_1 + A_n}{2} + \sum_{i=2}^{n-1} A_i\right) \quad or \quad V_{Surface-i} = \Delta x\left[A_2 + A_3 + \cdots + A_{n-1} + \frac{1}{2}(A_1 + A_n)\right] \quad (1)$$

where i is the counter, n is the number of stations, Δx is the distance between stations (m) and $V_{Surface-i}$ is the surface water storage volume (m^3).

Additionally, given the constant inflow rate into the furrow and the time it takes for water to reach each station, the total volume of water entering the furrow from the start of irrigation until the water reaches each station was calculated using Eq. 2.

$$V_{Inflow-i} = Q_o \times t_{x_i} \quad (2)$$

where $V_{Inflow-i}$ is the volume of inflow into the furrow from the start of irrigation until the water reaches the i-th station (m^3), Q_o is the flow rate entering the furrow at the upstream end (m^3/s), and t_{x_i} is the time taken for the water to reach the i-th station (min).

Subsequently, for each time step during the advance phase, the volume of infiltrated water was calculated by subtracting the surface-stored water volume from the inflow volume (Eq. 3).

$$V_{Infiltrated-i} = V_{Inflow-i} - V_{Surface-i} \quad (3)$$

Dividing the infiltrated water volume by the water advance distance along the field, the average depth of infiltrated water up to the i-th station per unit length of the furrow was obtained.

$$i_i = \frac{V_{Infiltrated-i}}{L_i} \quad and \quad L_i = \sum_{i=1}^{n} \Delta x_i \quad (4)$$

where i_i is the average depth of water infiltration at each stage of the water advance along the field (mm.m^2/m).

Additionally, the average opportunity time for water infiltration at each measured point during the advance phase was calculated using Eq.5.

$$\overline{t_{o-n}} = t_{x-n} - \frac{\sum_{i=1}^{n} t_{x-i}}{n} \quad (5)$$

where $\overline{t_{o-n}}$ is the average water infiltration opportunity time from the irrigation beginning to when water reaches the n-th station (min), t_{x-n} is the water advance time at the n-th station (min), and t_{x-i} is the water advance time at the i-th station (min).

Finally, for each experimental furrow in every irrigation event, the values of infiltrated water depth were obtained against the average infiltration opportunity time at each time step. Subsequently, using the nonlinear solver in Excel, the parameters for the Kostiakov (Eq. 6) and Kostiakov-Lewis (Eq. 7) infiltration models were derived.

$$i = kt^{a} \qquad (6)$$

$$i = kt^{a} + f_{o}t \qquad (7)$$

Method 2: Two-Point Method by Elliott and Walker (1982)

Elliott and Walker (1982)'s two-point method (TP) utilizes the volume balance equation to determine the coefficients of the Kostiakov (K) or Kostiakov-Lewis (KL) infiltration models (Eq. 8) (17). According to this method, during the water advance phase along the field, the total volume of inflow is equal to the sum of the surface-stored water and the infiltrated water volume.

$$V_x = \sigma_z k t_x^{\ a} = \frac{Q_o t_x}{x} - \sigma_y A_o - \frac{f_o t_x}{r+1} \qquad (8)$$

where A_o is the cross-sectional area at the upstream end of the field (m^2), x is the water advance distance from the upstream end of the field (m), t_x is the water advance time (min), and σ_y and σ_z are the shape factors.

Then, using Eq. 8 and the water advance data at two points-the mid- and end-point of the field-the two parameters of the Kostiakov or Kostiakov-Lewis infiltration models are determined as follows:

$$a = \frac{\log\left(V_L \big/ V_{\frac{L}{2}}\right)}{\log\left(t_L \big/ t_{\frac{L}{2}}\right)} \qquad \& \qquad k = \frac{V_L}{\sigma_z t_L^{\ a}} \qquad (9)$$

In the above equations, the value of σ_y is taken to be 0.77 (7) and the value of σ_z is calculated using the following equation (29).

$$\sigma_z = \frac{\int_0^L k t_x^{\ a} dx}{k t_L^{\ a}} = r\beta(r, a+1) \cong \frac{a + r(1-a) + 1}{(1+a)(1+r)} \qquad (10)$$

In this study, the WinSRFR software (version 5.1.1) was utilized to derive the parameters of the aforementioned equations using the two-point method.

Statistical Indices

Nash–Sutcliffe Efficiency (*NSE*)

To compare and determine the best-fitted infiltration model based on field data, the Nash-Sutcliffe Efficiency (*NSE*) statistical index was used. Its relationship is as Eq.11.

$$NSE = 1 - \frac{\sum_{i=1}^{n}\left(z_{i,O} - z_{i,P}\right)^2}{\sum_{i=1}^{n}\left(z_{i,O} - \overline{z_{i,O}}\right)^2} \qquad (11)$$

where $z_{i,O}$ is the experimental infiltration depth obtained from Eq. 4 (mm.m^2/m), $z_{i,P}$ is the calculated infiltration depth using the infiltration equations, and $\overline{z_{i,O}}$ is the averaged experimental infiltration depth obtained from Eq. 4.

The Nash-Sutcliffe index (*NSE*) ranges from negative infinity to unity (1.0). When the *NSE* value equals unity, there is a perfect match between the observed and predicted data (30). If the *NSE* value exceeds 0.75, the accuracy of the relationship is considered excellent, while values between 0.36 and 0.75 indicate that the predicted results are of moderate to good quality (31).

Coefficient of Determination (R^2)

The coefficient of determination (R^2) values from linear regression results that are close to unity signify high model accuracy. If $R^2 = x$, it means that x% of the total predicted values can be explained by a linear relationship between the predictions and the observations. This index evaluates linearity and is highly useful for improving model performance (32). Equation 12 was used to calculate this index.

$$R^2 = \left[\frac{\sum_{i=1}^{n} \left(z_{i,O} - \overline{z_{i,O}} \right)\left(z_{i,P} - \overline{z_{i,P}} \right)}{\sqrt{\sum_{i=1}^{n} \left(z_{i,O} - \overline{z_{i,O}} \right)^2 \times \sum_{i=1}^{n} \left(z_{i,P} - \overline{z_{i,P}} \right)^2}} \right]^2 \quad (12)$$

Mean Absolute Percentage Error (*MAPE*)

The Mean Absolute Percentage Error (*MAPE*) is a commonly used metric for evaluating the accuracy of predictive models. This index calculates the average percentage of absolute error between the actual values and the predicted values (mm.m^2/m), and $\overline{z_{i,O}}$ is the averaged experimental infiltration depth (mm.m^2/m). The equation for computing *MAPE* is as follows:

$$MAPE = \frac{100}{n} \sum_{i=1}^{n} \left| \frac{z_{i,O} - z_{i,P}}{z_{i,O}} \right| \quad (13)$$

The smaller the *MAPE* value, the better the accuracy of the predictive model. If the actual value is zero, the *MAPE* equation becomes undefined. *MAPE* may provide misleading results for datasets where the actual values are close to zero.

Boxplot Diagram

The boxplot is a widely used method for visualizing error distributions and comparing the performance of different models. This plot is analyzed based on four key metrics: the first quartile (Q1), the third quartile (Q3), the interquartile range (IQR), and the median. The first quartile (Q1) represents the value below which 25% of the errors fall, while the third quartile (Q3) represents the value below which 75% of the errors fall. Lower values for these two metrics indicate higher accuracy and better model performance. Among these, Q3 is particularly important as it covers a broader range of errors.

The interquartile range (IQR), which is the distance between Q1 and Q3, reflects the spread of error variations. This metric essentially determines the length of the box in the boxplot. A smaller IQR indicates more limited error variations and more favorable model performance. On the other hand, the median, represented by the central line of the box, indicates higher model accuracy when it is closer to zero.

By combining the median and IQR metrics, the symmetry of the error distribution can be evaluated. If the central line of the box is close to the center of the box, the error distribution is more symmetrical. This symmetry suggests that the errors are evenly distributed around the median value.

Results and Discussion

Figure 3 illustrates the infiltrated water depth in the furrows plotted against the average infiltration opportunity time using the infiltration equation derived from the all advance time steps method. The infiltration curves fitted to the Kostiakov and Kostiakov-Lewis models are also presented. Table 3 provides the coefficients of the infiltration relationships derived from both methods, including all advance time steps and the two-point method. Table 4 presents the accuracy of these fitted infiltration relationships based on field data. According to this table, the accuracy of infiltration relationships derived from both methods is satisfactory for all furrows. Based on the *NSE* index, the accuracy of the Kostiakov relationship is very good accuracy (greater than 0.75) in two furrows and good to moderate accuracy (between 0.3 and 0.75) in the remaining three furrows. However, the *NSE* index for the Kostiakov-Lewis relationship exceeded or approached to 0.75 in all furrows. Furthermore, based on the *MAPE* index, the Kostiakov-Lewis relationship demonstrated better agreement with the field data than the Kostiakov relationship.

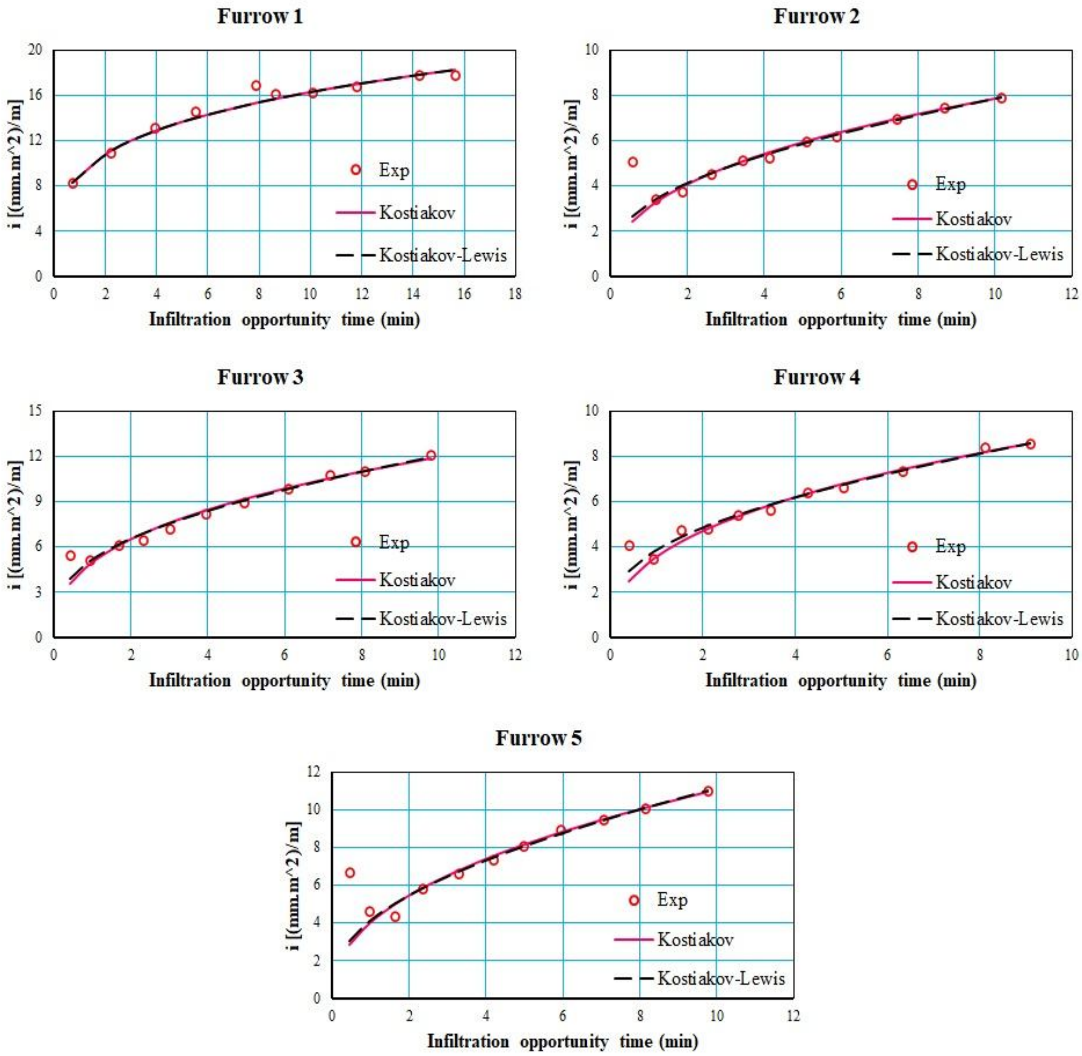

Figure 3. Infiltration depths, average infiltration time opportunity, and fitted infiltration relationships.

Table 3. Kostiakov and Kostiakov-Lewis infiltration relations extracted from all advance time stations and the two-point method.

Infiltration relationship	Infiltration relationship coefficients	Furrow No.				
		1	2	3	4	5
Method 1 (Kostiakov)	a	0.254	0.407	0.375	0.396	0.442
	$k\left(mm.m^2/m.h^a\right)$	25.666	16.287	23.384	18.049	24.471
Method 1 (Kostiakov Lewis)	a	0.255	0.319	0.302	0.276	0.355
	$k\left(mm.m^2/m.h^a\right)$	25.695	11.310	17.069	11.420	16.810
	$f_o\left(mm.m^2/m.h\right)$	-	8.792	12.778	11.834	13.422
Two-point method (Kostiakov)	a	0.1710	0.391	0.567	0.742	0.492
	$k\left(mm.m^2/m.h^a\right)$	25.964	17.123	27.871	28.832	22.913
Two-point method (Kostiakov Lewis)	a	-	0.389	0.474	0.741	0.490
	$k\left(mm.m^2/m.h^a\right)$	-	17.027	18.577	28.655	22.751
	$f_o\left(mm.m^2/m.h\right)$	-	8.792	12.778	11.834	13.422

Table 4. Accuracy of fitting the infiltration relationships fitted from field data based on statistical indicators.

Infiltration relationship	Statistical Indicators	Furrow No.				
		1	2	3	4	5
Method 1 (Kostiakov)	*MAPE* (%)	2.46	6.45	5.46	5.74	8.59
	R^2	0.968	0.776	0.948	0.940	0.791
	NSE	0.963	0.657	0.931	0.897	0.672
Method 1 (Kostiakov- Lewis)	*MAPE* (%)	2.44	5.99	4.35	5.28	7.95
	R^2	0.967	0.793	0.960	0.951	0.805
	NSE	0.962	0.706	0.950	0.940	0.712

Additionally, using the aforementioned infiltration relationships and the average field opportunity time during each irrigation event, the infiltrated water volume (calculated) in the field was determined. Table 5 presents the actual and calculated infiltrated water volumes in the furrows, as well as the *MAPE* index values of the infiltration relationships. The average *MAPE* for the Kostiakov and Kostiakov-Lewis infiltration equations derived from the first method was 44.3% and 12.6%, respectively, while for the two-point method, it was 28.2% and 26.1%, respectively. In both methods, the Kostiakov-Lewis infiltration equation demonstrated higher accuracy compared to the Kostiakov equation, which aligns with previous findings. Furthermore, the lowest error was associated with the Kostiakov-Lewis equation derived from the first method, while the highest error was attributed to the Kostiakov equation derived from the same method. This highlights the importance of the equation form in fitting methods or approaches where infiltration equation coefficients are determined using the least squares error minimization. In the two-point method, the coefficient derivation process is identical for both equations, with the only difference being the addition of the basic infiltration rate parameter for the Kostiakov-Lewis equation. As a result, the error values for both equations are nearly identical. Typically, if the error of infiltration equations is less than 10% (7), there is no need to modify the equation when plotting the infiltrated water depth profile. In the current study, the Kostiakov-Lewis equation derived from the first method for furrows 2 and 5, and the Kostiakov and Kostiakov-Lewis equations derived from the two-point method for furrow 4, do not require modification. Elliott and Walker (1982) reported that infiltration equation derivation methods that account for dynamic field conditions, unlike point-based infiltration measurement methods, do not require modification (17). However, as observed, the equations obtained from both methods require adjustments, which is consistent with the findings of Maroufouur et al. (2017) (33). Another important consideration is the advance time and cutoff time in the experimental furrows. In all experimental furrows except the first furrow, the advance time is significantly shorter than the cutoff time (Table 1), and the mentioned infiltration equations are solely based on infiltration during the advance phase, which corresponds to short infiltration times. Therefore, their accuracy in predicting infiltration over longer durations, such as the average opportunity time during the entire irrigation event, may be limited. Additionally, evaluating the accuracy of the equations based on the average field opportunity time and the average actual infiltration depth in the field may not be an ideal indicator. In other words, the point in the field where the average opportunity time occurs does not necessarily coincide with the point where the average actual infiltration depth occurs. Thus, this point is merely a hypothetical reference used in surface irrigation system evaluations to adjust the infiltration equation and align the average infiltration depths of the stations with the average actual infiltration depth in the field.

Table 5. The amount of actual and calculated infiltrated water volume by different infiltration relationships and their accuracy.

Parameter	Type of infiltration relationship	Furrow No.				
		1	2	3	4	5
Actual infiltrated water volume (m^3)	-	2.803	4.584	9.857	10.503	7.801
Calculated infiltrated water volume (m^3)	Method 1 (Kostiakov)	2.471	2.794	3.909	3.737	4.239
	Method 1 (Kostiakov-Lewis)	2.474	5.014	7.570	9.291	7.234
	Two-point Method (Kostiakov)	2.458	2.877	6.045	11.235	4.220
	Two-point Method (Kostiakov-Lewis)	-	2.908	8.383	11.269	4.257
MAPE (%)	Kostiakov	11.88	39.05	60.35	64.42	45.66
	Method 1 (Kostiakov-Lewis)	11.77	9.37	23.21	11.54	7.27
	Two-point method (Kostiakov)	12.30	37.2	38.7	7.0	45.9
	Two-point method (Kostiakov-Lewis)	-	36.6	15.0	7.3	45.4

During the operational phase, irrigation management occurs at varying depths and durations, necessitating that the derived infiltration relationships maintain acceptable accuracy across different infiltration times. Therefore, the accuracy of the relationships was evaluated using data on the average infiltration depth during all irrigation times (irrigation start time to the cutoff time) and the average field opportunity times. The evaluation of these relationships in this section was conducted using boxplot diagrams. Figure 4 presents the evaluation results of the derived infiltration relationships.

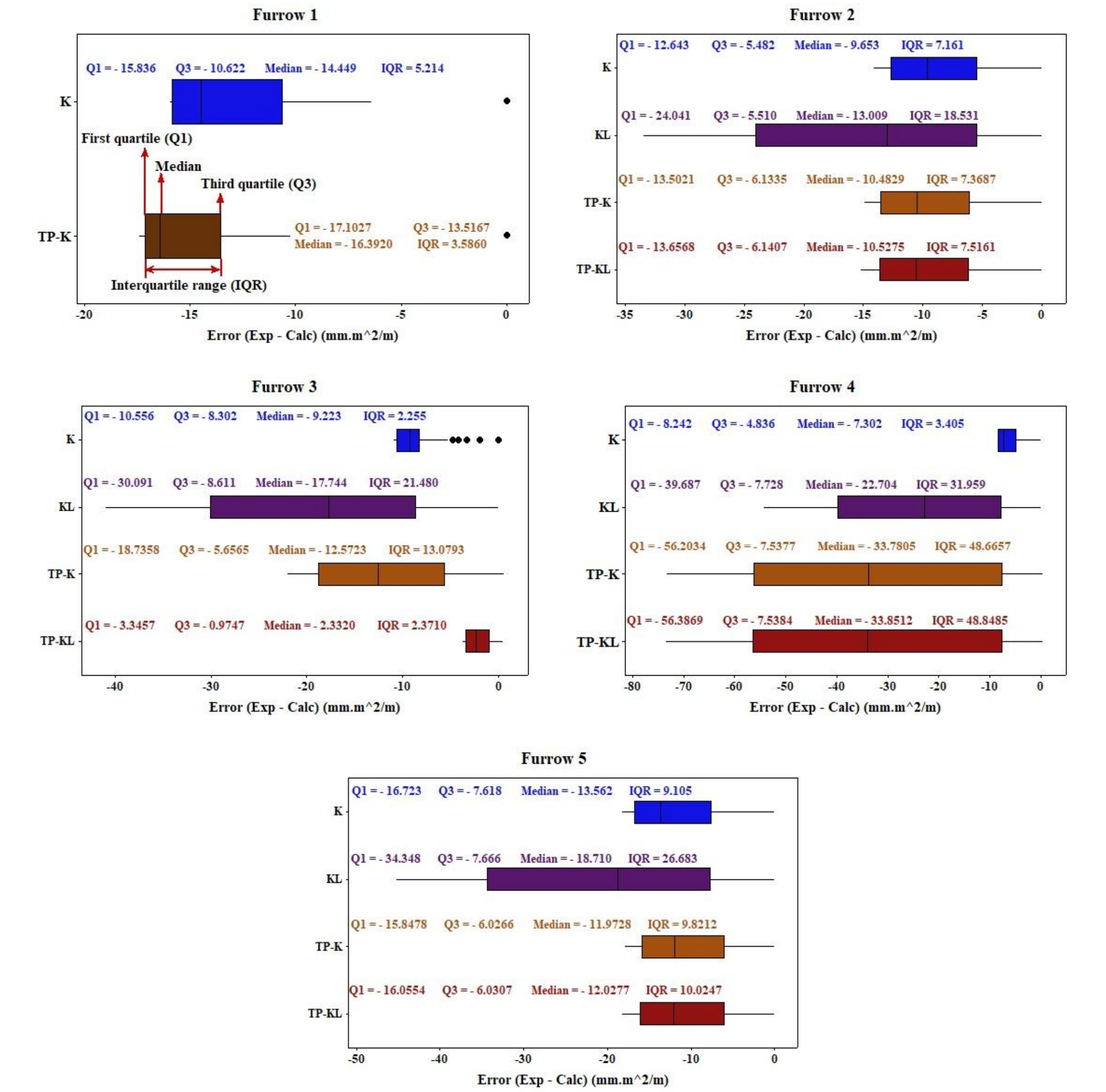

Figure 4. Error boxplot of different methods in estimating the infiltrated water depth during irrigation time.

According to Figure 4, with the boxes positioned to the left of the horizontal axis, it is evident that all equations have overestimated the infiltration depth. In terms of the accuracy of the equations, the results are as follows:

- In the first furrow, where only the Kostiakov equations were compared, the Kostiakov equation derived from the first method shows slightly higher accuracy.
- In the second furrow, the Kostiakov equation from the first method ranks first, followed closely by the Kostiakov and Kostiakov-Lewis equations from the two-point method in second place.
- In the third furrow, the Kostiakov-Lewis equation from the two-point method ranks first, followed by the Kostiakov equation from the first method.
- In the fourth furrow, the Kostiakov equation from the first method ranks first, with the Kostiakov-Lewis equation from the first method significantly behind in second place.
- In the fifth furrow, the Kostiakov and Kostiakov-Lewis equations from the two-point method rank first, followed closely by the Kostiakov equation from the first method.

Overall, in all furrows except the third furrow, the Kostiakov equation derived from the first method exhibits equal or greater accuracy compared to the other equations. In the third furrow, the error range of the top-rank two equations is similar, but the Kostiakov-Lewis equation from the two-point method has a lower median than the Kostiakov equation from the first method. Additionally, in all furrows except the third, the results of the infiltration equations from the two-point method are nearly identical.

The results obtained from this analysis contradict those derived from the hypothetical point of average actual infiltration depth and average field opportunity time. In other words, equations with minimal error at the mentioned hypothetical point are not necessarily the most accurate infiltration equations. The Kostiakov equation derived from the first method, with a median error of 7.3 to 14.5 (mm.m^2/m), demonstrates satisfactory accuracy in estimating the infiltrated water depth. In contrast, the Kostiakov-Lewis equation derived from the first method, with the highest median error of 13.0 to 22.7 (mm.m^2/m), exhibits the lowest accuracy.

Conclusion

This study introduced a novel all-time-step approach for determining infiltration parameters in surface irrigation, leveraging continuous water depth measurements from every advance phase along the field. The results demonstrated that the Kostiakov-Lewis equation derived via the all-time-step method (Method 1) achieved superior accuracy in estimating average infiltration depth, with a *MAPE* of 12.6%, significantly lower than the 26.1% achieved by the traditional two-point method (Method 2). The Kostiakov-Lewis model consistently outperformed the Kostiakov model during the advance phase, highlighting the importance of incorporating a steady-term parameter for short-term infiltration dynamics. However, the Kostiakov model showed better performance during the storage phase, suggesting its adaptability to prolonged infiltration scenarios.

The key innovation of this study lies in the all-time-step approach, which captures dynamic infiltration patterns often overlooked by traditional methods. By integrating data from multiple points during the advance phase, this method provides a more comprehensive and accurate estimation of empirical infiltration parameters, leading to improved calibration of the Kostiakov and Kostiakov-Lewis equations. This result has significant implications for irrigation planning, particularly in water-scarce regions, as it enables improved irrigation scheduling, optimized water-use efficiency, and enhanced support for sustainable agricultural practices.

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Data Availability Statement

The data used in this study were sourced from Ramsey’s (1976) Master's thesis (a graduate student at the University of Arizona). All data are publicly accessible either by downloading Ramsey’s (1976) thesis or upon request from the authors of this study.

Author Contributions

First author: Data extraction, visualization (figures and charts), manuscript drafting, and revision.
Second author: Data analysis.
Third author: Conceptualization, manuscript editing, and research supervision.

Ethical Principles

All authors confirm that they have observed ethical principles in preparing and publishing this study.

Conflicts of Interest

The authors declare no conflicts of interest.

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